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The Scarcity of Interleaved Practice in Mathematics Textbooks

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Abstract

A typical mathematics assignment consists of a block of problems devoted to the same topic, yet several classroom-based randomized controlled trials have found that students obtain higher test scores when most practice problems are mixed with different kinds of problems – a format known as interleaved practice. Interleaving prevents students from safely assuming that each practice problem relates to the same skill or concept as the previous problem, thus forcing them to choose an appropriate strategy on the basis of the problem itself. Yet despite the efficacy of interleaved practice, blocked practice predominates most mathematics textbooks. As an illustration, we examined 13,505 practice problems in six representative mathematics texts and found that only 9.7% of the problems were interleaved. This translates to only one or two interleaved problems per school day. In brief, strong evidence suggests that students benefit from heavy doses of interleaved practice, yet most mathematics texts provide scarcely any.

Keywords: mathematics, textbook, interleaved, mixed, practice

A majority of the effort that students devote to mathematics learning is spent solving practice problems, and the benefit of this effort depends crucially on a seemingly innocuous feature – the order in which the practice problems appear. Whereas the usual practice assignment consists of a block of problems devoted to the same skill or concept, randomized controlled trials conducted in the laboratory and classroom have shown that students score higher on delayed tests if most of their practice problems are interleaved so that consecutive problems cannot be solved by the same strategy. In light of this evidence, we maintain that mathematics textbooks should provide an adequate number of interleaved problems, simply because the student textbook is the primary source of practice problems in many mathematics classes. Here we provide a brief review of the relevant research, and we present an analysis of the prevalence of interleaved and blocked practice in six representative mathematics textbooks.

Blocked Practice

A group of practice problems is *blocked* if every problem relates to the same skill or concept. For instance, a lesson on ratios might be followed by a block of a dozen ratio problems. In some assignments, the blocking is salient because every problem shares the same instruction (Fig. 1). More often, though, the blocking is less obvious. For instance, a lesson on circumference might be followed by a set of problems that are ostensibly diverse yet nevertheless devoted to circumference (Fig. 2). In addition, many textbooks provide periodic review assignments that consist of small blocks of practice problems. For instance, chapter review assignments often consist of a few problems about the first lesson in the chapter, followed by a few problems about the second lesson, and so forth. Small blocks also commonly appear in assignments providing so-called spiral review or mixed review (Fig. 3).

Despite the ubiquity of blocked practice, we are not aware of any studies showing that students benefit from working many similar problems in a single session. We know of two laboratory experiments that manipulated the number of similar practice problems worked in immediate succession (for example, 2 vs. 4 problems, or 3 vs. 9 problems), and neither study

found a discernible effect on test scores (Rohrer & Taylor, 2006, 2007). Similarly, in numerous studies of verbal learning, subjects who studied words until they reached a criterion of two or three correct responses and then continued to study the same material *immediately* afterwards (rather than quit) showed only a small and fleeting increase in test scores (see meta-analysis by Driskell, Willis, & Copper, 1992). On the other hand, these kinds of studies do not rule out the possibility that students might benefit from working at least a *few* problems of the same kind when they first encounter a new skill or concept. This immediate repetition might reduce the intellectual demands of the problems, as each problem acts as a worked example for the next one, and this kind of scaffolding might be optimal when students try to solve a new kind of problem (e.g., Kotovsky, Hayes, & Simon, 1985; Paas & van Merriënboer, 1994; Sweller, van Merriënboer, & Paas, 1998). Still, no evidence suggests that students should trudge through long blocks of practice problems.

Furthermore, the scaffolding provided by blocked practice has a downside, as illustrated by the following example.

A bug crawls 24 cm west and then 7 cm north. How far is the bug from where it started?

To solve this problem, students must first infer that they need to use the Pythagorean Theorem $(24^2 + 7^2 = c^2, \text{ so c} = 25 \text{ cm})$. Yet this inference is unnecessary when the problem appears as part of a blocked assignment, especially if the block immediately follows a lesson on the Pythagorean Theorem. Put another way, the solution of a mathematics problem requires students to both *choose* and *execute* a strategy, yet blocking often allows students to infer an appropriate strategy for a problem before they read the problem. In effect, blocked practice denies students the opportunity to learn how to choose a strategy on the basis of the problem itself, which is what students must do when they sit for a cumulative exam.

Interleaved Practice

In the complement to blocked practice known as *interleaved practice*, problems are arranged so that consecutive problems do not relate to the same skill or concept, thereby forcing students to choose a strategy and not only execute it. Choosing a strategy is not always trivial because two problems that look alike might demand different strategies. For instance, arithmetic students might struggle to determine whether a problem requires addition or subtraction (*If Ben ate 8 cookies and now has 3 cookies, how many did he begin with?*). In Calculus, many integral problems look alike yet require different strategies (e.g., ∫exed is solved by substitution, yet ∫xexdx requires integration by parts). In simplest terms, interleaved practice provides students with an opportunity to choose an appropriate strategy on the basis of the problem itself, which is exactly what students are expected to learn. In this sense, interleaved practice is an instantiation of the most fundamental principle of learning: the practice of a task improves the performance of that task.

An interleaved assignment can take on various guises. Most commonly, each problem is unrelated to every other problem in the assignment. Alternatively, consecutive problems can be superficially similar yet relate to easily confused concepts, such as a problem on direct variation followed by one on inverse variation. In yet another configuration, every problem is based on the same scenario, such as a data histogram followed by problems on the mean, median, standard deviation, and interquartile range. Sample interleaved assignments are shown in Fig. 4.

Apart from any benefits derived from interleaving per se, the regular use of interleaved practice throughout a course inherently increases the degree to which each kind of problem is distributed or *spaced* throughout the course. For example, whereas most of the parabola problems in a heavily-blocked textbook might be concentrated in one or two assignments, the same parabola problems would be distributed across one or two dozen assignments in a heavily-interleaved textbook. A greater degree of spacing improves scores on delayed tests, and this effect is one of the largest and most robust effects in learning (for reviews, see

Carpenter, Cepeda, Rohrer, Kang, & Pashler, 2012; Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013). Most spacing experiments have taken place in the laboratory with verbal materials, but each of a recent spate of studies found a spacing effect in mathematics classrooms (Barzagar Nazari & Ebersbach, 2019; Chen, Castro-Alonso, Paas, & Sweller, 2018; Hopkins, Lyle, Hieb, & Ralston, 2016; Lyle, Bego, Hopkins, Hieb, & Ralston, in press; Schutte, Duhon, Solomon, Poncy, Moore, & Story, 2015). In summary, interleaved mathematics assignments have two potentially useful features: different kinds of problems are mixed together within the same assignment (which can improve strategy choice), and problems of the same kind are spaced across different assignments (which can improve long-term retention).

Studies Comparing Interleaved and Blocked Practice

More than a dozen randomized controlled studies have compared the efficacy of interleaved and blocked mathematics practice, and each found that a greater dose of interleaved practice produced higher scores on the final test. This *interleaving effect* was first obtained in the laboratory (LeBlanc & Simon, 2008; Mayfield & Chase, 2002; Rohrer & Taylor, 2007; Taylor & Rohrer, 2010), and two recent sets of laboratory studies replicated the effect while examining more sophisticated questions (Foster, Mueller, Was, Rawson, & Dunlosky, 2019; Sana, Yan, & Kim, 2017). Other interleaving studies took place in the classroom. In a study of fifth- and sixth-grade students using an online tutor to learn about fractions, interleaving produced higher scores on tests given immediately and again one week later (Rau, Aleven, & Rummel, 2013). In a similar study, seventh grade students who completed an interleaved online review (rather than a blocked one) scored slightly higher on a test given 2-5 days later (Ostrow, Heffernan, Heffernan, & Peterson, 2015). Finally, in two studies with longer time intervals, seventh-grade students received mostly interleaved or mostly blocked practice over several months before completing a business-as-usual review assignment followed one month later by an unannounced test, and the higher dose of interleaving produced much higher test scores

(Rohrer, Dedrick, Hartwig, & Cheung, in press; Rohrer, Dedrick, & Stershic, 2015). In short, interleaved mathematics practice is supported by a variety of ecologically-valid studies.

The literature does not, however, specify exactly what proportion of practice problems should be interleaved. In fact, the optimal proportion is certainly unknowable, as it likely depends on several factors, including student characteristics such as proficiency and motivation, problem features such as novelty and difficulty, and temporal parameters such as the time intervals between assignments. Nevertheless, each of the aforementioned studies found that test scores were higher when practice problems were mostly interleaved rather than mostly blocked, and thus the evidence conservatively suggests that mathematics students should work at least several interleaved problems each school day.

Interleaved practice is also practical. It can be done in class or at home, with or without a computer. The curriculum need not be altered, and teachers need not revise their tutorials. Interleaved practice is also known to be viable because mathematics teachers have long assigned interleaved review problems prior to cumulative final exams and high-stakes tests. Some evidence also suggests that interleaved practice has teacher buy-in. In one of the interleaving studies described above (Rohrer et al., in press), participating teachers completed an anonymous survey after they finished the study (but before they knew the purpose or results of the study), and nearly every teacher endorsed interleaved practice on a variety of measures of efficacy and feasibility. In one notable exception, however, most of the teachers reported that a practice problem was generally easier for students when it appeared as part of a block rather than as part of an interleaved assignment, presumably because of the scaffolding provided by blocking. On the whole, though, interleaved practice appears to be a feasible classroom intervention, and the evidence for its efficacy is broad and ecologically valid. For these reasons, numerous learning researchers have endorsed interleaved mathematics practice in outlets intended for teachers and laypeople (e.g., Deans for Impact, 2015; Dunlosky, 2013; Pan, 2015; Roediger & Pyc, 2012; Willingham, 2014).

Textbooks

In view of the large and robust benefits of interleaved practice, we argue that mathematics textbooks should include an adequate number of interleaved problems because the student textbook is the primary source of learning material for most students (e.g., Blazar et al., 2019). To be sure, teachers can write their own assignments or draw materials from the internet and other resources, but these alternatives require experience, time, and sometimes additional costs (e.g., photocopying). Put bluntly, the school-provided mathematics textbook is a primary and costly classroom resource, and thus its design and content should be evidence-based.

Yet nearly every mathematics textbook we have seen is predominately blocked, and we have come across dozens of textbooks during our years as mathematics learning researchers. It seems to us, in fact, that blocking dominates mathematics textbooks at all levels, from kindergarten through college, though our experience is admittedly limited to textbooks sold in the United States. To test our anecdotal observations, we measured the prevalence of blocked and interleaved practice in a selection of mathematics textbooks that are popular in the U.S.

We restricted our analyses to textbook series that span middle school (grades 6-8), primarily because nearly all of our research has taken place in middle schools. The number of middle school math series available in the U.S. is unclear, as publishers frequently add or discontinue titles, but one recent report lists 27 middle school math series (edreport.org, 2019). We do not know the market share of any of these texts because such data are reportedly not available (e.g., Polikoff, 2018). However, we believe that the market is dominated by fewer than a dozen titles, and thus we sought to limit our sample to the most widely-used textbooks in order to avoid including obscure texts that few students use. Toward this aim, we first asked a research assistant (who was blind to the research question) to look for lists of approved middle school mathematics textbooks for the 10 most populous states in the U.S., and she was able to find the adoption lists for four of the states (California, Texas, Florida, and North Carolina). Next, we created a list of texts that appeared on *each* of the four adoption lists (excluding digital-only

texts), and then we asked the research assistant to obtain as many of these titles as she could. In short, we did not cherry pick the texts.

The selection included six textbook series: *Big Ideas Math* (Larson & Boswell, 2014), *Connected Mathematics* (Lappan, Phillips, Fey, & Friel, 2014), *Glencoe Math: Built to the Common Core* (Carter et al., 2015), *Go Math* (Burger et al., 2014), *Holt McDougal Mathematics* (Bennet et al., 2012), and *SpringBoard Mathematics* (Allwood et al., 2014). None of us has a current or previous affiliation with any of these textbooks or the companies that publish them. We examined only the seventh grade text in each series, although textbooks within the same series typically have similar characteristics.

The six textbooks include 13,505 practice problems. We classified a problem as blocked if it satisfied either of two criteria: (1) the problem was the first one in an assignment devoted solely to the immediately preceding lesson (e.g., the first ratio problem in a set of ratio problems following a lesson on ratios), or (2) the problem was based on the same skill or concept as the immediately preceding problem. A problem that did not clearly satisfy either of these criteria was classified as interleaved. During the classification process, we struggled to classify some problems as either blocked or interleaved (e.g., a problem about two concepts following a problem related to just one of the two concepts), and we labeled these problems as ambiguous (9.7% of the problems). One rater classified every problem, and a second rater classified a subset of 1,792 problems (intraclass correlation coefficient = .97).

We found that each of the six textbooks is heavily blocked, and even the review assignments in each text are moderately blocked (Fig. 5). On average, each textbook includes 1814 blocked problems (range = 701-2,939) and 219 interleaved problems (range = 111-392). This translates to more than eight blocked problems for every interleaved problem, and only one or two interleaved problems per school day in a typical school year.

Of course, we cannot conclude that interleaved practice is scarce in every math textbook because our sample excluded numerous textbooks. Still, we sought to create a representative

sample, and we suspect that our sample comprises a majority of the market. Moreover, blocked practice predominates *every* textbook in our sample – not just a *majority* of the texts. If an urn includes a few dozen marbles, and *each* of the first six drawn marbles is red, most of the remaining marbles are probably red, too. Another limitation of our analysis is that we excluded so-called consumable workbooks, which provide white space for students to write their solutions, and which are increasingly supplementing or replacing traditional textbooks in many mathematics classrooms. However, the consumable workbooks we have seen provide fewer interleaved problems than do textbooks, though we have not conducted a formal analysis like the textbook analysis presented here. Finally, we emphasize that the present analysis leaves open the possibility that some teachers might be providing students with interleaved assignments drawn from sources other than the school-assigned textbook.

A Notable Exception

Although blocked practice predominates nearly every mathematics textbook we have seen, interleaved practice is a hallmark of the controversial Saxon math series (K-12). As an illustration, we examined every practice problem in the seventh-grade Saxon text (Hake, 2012) and found that it included 1,491 blocked problems and 3,016 interleaved problems. That translates to more than a dozen interleaved problems for each school day in the year. An excerpt of a typical Saxon assignment is shown in Fig. 4.

Yet the Saxon math series no longer appears on any of the textbook adoption lists that we could find, possibly because its publisher did not update the series so that it aligns with the mathematics curriculum recently adopted throughout most of the United States (known as the Common Core). We suspect that this business decision was due at least partly to widespread criticism by mathematics learning researchers who have complained that Saxon emphasizes procedures and algorithms at the expense of conceptual understanding (for a discussion of the controversy, see Jacob, 2001). On this contentious issue the present commentary is agnostic, though we would argue that, *all else equal*, interleaved practice demands greater conceptual

understanding than does blocked practice. Importantly, however, the criticism of Saxon, whether founded or not, is orthogonal to its use of interleaved practice. We also emphasize that we are advocating for interleaved practice per se and not necessarily for Saxon mathematics texts. Indeed, the efficacy of a Saxon text – and any other mathematics textbook – depends on numerous features other than the prevalence of interleaved practice, such as the choice of content, organization of materials, difficulty of problems, number of problems, and so forth. Nevertheless, the heavily-interleaved assignments within Saxon are supported by empirical evidence. (None of us has a current or previous affiliation with Saxon or its publisher.)

Recommendations

In light of the efficacy of interleaved practice and its scarcity in most mathematics texts, we urge creators of textbooks and other instructional materials to add a sufficient number of interleaved problems to the next edition. This can be accomplished by merely rearranging a portion of the blocked practice problems appearing in the current edition, without altering the lessons or the organization of the textbook. Such an updating should not affect the price of the next edition, meaning that interleaved practice could be implemented at no cost to students or taxpayers. These kinds of textbook revisions are hardly novel, as mathematics textbooks have changed substantially over the last several decades in response to the ever-changing decision criteria used in the textbook selection process. We are simply recommending that interleaved practice be added to the list of criteria.

Finally, we recommend that mathematics teachers avoid assigning long blocks of practice problems. No direct evidence suggests that students benefit from solving more than a few problems of the same kind in immediate succession, whereas interleaved assignments are supported by numerous classroom-based, randomized controlled studies. If the classroom textbook includes few interleaved problems, we suggest that teachers create interleaved assignments, perhaps by simply assigning one problem from each of a dozen or so blocked

assignments in the students' textbook. Ultimately, though, we hope that mathematics students will have access to learning materials that are aligned with the empirical evidence.

References

- Allwood, M., Bullard, F., Chumas, M., Fritz, K., Harris, S., Humphrey, M., Kotz, B., Olsen, C., Peck, R., Sheets, K., Sukow, A., & Tate, S. (2014). *SpringBoard Mathematics: Course 2*. New York, NY: College Board.
- Barzagar Nazari, K., & Ebersbach, M. (2019). Distributing mathematical practice of third and seventh graders: Applicability of the spacing effect in the classroom. *Applied Cognitive Psychology*, 33, 288-298.
- Bennett, J., Burger, E., Chard, D., Hall, E., Kennedy, P., Renfro, F., Roby, T., Scheer, J., & Waits, B. K. (2012). *Holt McDougal Mathematics: Grade 7.* Orlando, FL: Houghton Mifflin Harcourt.
- Blazar, D., Heller, B., Kane, T., Polikoff, M., Staiger, D., Carrell, S.,...& Kurlaender, M. (2019). Learning by the book: Comparing math achievement growth by textbook in six Common Core states. Research Report. Cambridge, MA: Center for Education Policy Research, Harvard University.
- Burger, E. B., Dixon, J. K., Kanold, T. D., Larson, M. R., Leinwand, S. J., & Sandoval-Martinez, M. E. (2014). *Go Math: Middle School*, Grade 7. Orlando, FL: Houghton Mifflin Harcourt.
- Carpenter, S. K, Cepeda, N. J., Rohrer, D., Kang, S. H. K., & Pashler, H. (2012). Using spacing to enhance diverse forms of learning: Review of recent research and implications for instruction. *Educational Psychology Review*, *24*, 369-378.
- Carter, J. A., Cuevas, G. J., Day, R., Malloy, C. E., Kersaint, G., Reynosa, M., Silbey, R., & Vielhaber, K. (2015). *Glencoe Math: Built to the Common Core*. Columbus, OH: McGraw-Hill Education.
- Chen, O., Castro-Alonso, J. C., Paas, F., & Sweller, J. (2018). Extending cognitive load theory to incorporate working memory resource depletion: Evidence from the spacing effect. *Educational Psychology Review*, *30*, 483-501.
- Deans for Impact (2015). The science of learning. Austin, TX: Deans for Impact.
- Driskell, J. E., Willis, R. P., & Copper, C. (1992). Effect of overlearning on retention. *Journal of Applied Psychology*, 77, 615-622.
- Dunlosky, J. (2013). Strengthening the student toolbox: Study strategies to boost learning. *American Educator*, Fall, 12-21.
- Dunlosky, J., Rawson, K. A., Marsh, E. J., Nathan, M. J., & Willingham, D. T. (2013). Improving students' learning with effective learning techniques: Promising directions from cognitive and educational psychology. *Psychological Science in the Public Interest*, *14*, 4-58.
- EdReports.org (2019). Compare materials. Retrieved October 17, 2019, from www.edreports.org.

- Foster, N. L., Mueller, M. L., Was, C., Rawson, K. A., & Dunlosky, J. (2019). Why does interleaving improve math learning? The contributions of discriminative contrast and distributed practice. *Memory & Cognition*, *47*, 1088-1101.
- Hake, S. (2012). Saxon Math: Course 2. Orlando, FL: Houghton Mifflin Harcourt.
- Hopkins, R. F., Lyle, K. B., Hieb, J. L., & Ralston, P. A. (2016). Spaced retrieval practice increases college students' short-and long-term retention of mathematics knowledge. *Educational Psychology Review*, *28*(4), 853-873.
- Jacob, B. (2001). Implementing standards: The California mathematics textbook debacle. *Phi Delta Kappan*, *83*(3), 264-272.
- Kotovsky, K., Hayes, J. R., & Simon, H. A. (1985). Why are some problems hard? Evidence from Tower of Hanoi. *Cognitive Psychology*, *17*(2), 248-294.
- Lappan, G., Phillips, E., Fey, J., & Friel, S. (2014). *Connected Mathematics, Grade 7.* Boston, MA: Pearson Education, Inc.
- Larson, R., & Boswell, L. (2014). *Big Ideas Math: A Common Core Curriculum (Red Edition)*. Erie, PA: Big Ideas Learning, LLC.
- Le Blanc, K., & Simon, D. (2008, November). *Mixed practice enhances retention and JOL accuracy for mathematical skills*. Paper presented at the 49th Annual Meeting of the Psychonomic Society, Chicago, IL.
- Lyle, K. B., Bego, C. R., Hopkins, R. F., Hieb, J. L., & Ralston, P. A. S. (in press). How the amount and spacing of retrieval practice affect the short- and long-term retention of mathematics knowledge. *Educational Psychology Review*.
- Mayfield, K. H., & Chase, P. N. (2002). The effects of cumulative practice on mathematics problem solving. *Journal of Applied Behavior Analysis*, *35*, 105-123.
- Ostrow, K., Heffernan, N., Heffernan, C., & Peterson, Z. (2015). Blocking vs. interleaving: Examining single-session effects within middle school math homework. *Artificial Intelligence in Education* (pp. 338-347). Springer, Cham.
- Paas, F., & van Merriënboer, J. J. G. (1994). Variability of worked examples and transfer of geometrical problem-solving skills: A cognitive-load approach. *Journal of Educational Psychology*, 86, 122-133.
- Pan, S. C. (August 4, 2015). The interleaving effect: Mixing it up boosts learning. *Scientific American Mind*.
- Polikoff, M. (2018). The challenges of curriculum materials as a reform lever. *EducationNext*. *Retrieved from www.educationnext.org/challenges-curriculum-materials-reform-lever*
- Rau, M. A., Aleven, V., & Rummel, N. (2013). Interleaved practice in multi-dimensional learning tasks: Which dimension should we interleave? *Learning and Instruction*, 23, 98-114.
- Roediger, H. L., & Pyc, M. A. (2012). Inexpensive techniques to improve education: Applying cognitive psychology to enhance educational practice. *Journal of Applied Research in Memory and Cognition*, 1(4), 242-248.

- Rohrer, D., & Taylor, K. (2006). The effects of overlearning and distributed practice on the retention of mathematics knowledge. *Applied Cognitive Psychology*, 20, 1209-1224.
- Rohrer, D., & Taylor, K. (2007). The shuffling of mathematics practice problems boosts learning. *Instructional Science*, *35*, 481-498.
- Rohrer, D., Dedrick, R. F., & Stershic, S. (2015). Interleaved practice improves mathematics learning. *Journal of Educational Psychology*, *107*, 900-908.
- Rohrer, D., Dedrick, R. F., Hartwig, M. K., & Cheung, C. N. (2020). A randomized controlled trial of interleaved mathematics practice. *Journal of Educational Psychology*, *112*, 40-52.
- Sana, F., Yan, V. X., & Kim, J. A. (2017). Study sequence matters for the inductive learning of cognitive concepts. *Journal of Educational Psychology*, *109*, 84-98.
- Schutte, G. M., Duhon, G. J., Solomon, B. G., Poncy, B. C., Moore, K., & Story, B. (2015). A comparative analysis of massed vs. distributed practice on basic math fact fluency growth rates. *Journal of School Psychology*, *53*, 149-159.
- Sweller, J., Van Merriënboer, J. J. G., & Paas, F. G. W. C. (1998). Cognitive architecture and instructional design. *Educational Psychology Review*, *10*, 251-296.
- Taylor, K., & Rohrer, D. (2010). The effect of interleaving practice. *Applied Cognitive Psychology*, 24, 837-848.
- Willingham, D. T. (2014). Strategies that make learning last. *Educational Leadership*, 72(2), 10-15.

Multiply. Write each answer in simplest form.

Holt McDougal (p. 124)

1.
$$-8 \cdot \frac{3}{4}$$

2.
$$\frac{2}{3} \cdot \frac{3}{5}$$

1.
$$-8 \cdot \frac{3}{4}$$
 2. $\frac{2}{3} \cdot \frac{3}{5}$ 3. $\frac{1}{4} \cdot \left(-\frac{2}{3}\right)$ 4. $\frac{3}{5} \cdot (-15)$ 5. $4 \cdot 3\frac{1}{2}$ 6. $\frac{4}{9} \cdot 5\frac{2}{5}$ 7. $1\frac{1}{2} \cdot 1\frac{5}{9}$ 8. $2\frac{6}{7} \cdot (-7)$

4.
$$\frac{3}{5} \cdot (-15)$$

5.
$$4 \cdot 3^{\frac{1}{2}}$$

6.
$$\frac{4}{9} \cdot 5\frac{2}{5}$$

7.
$$1\frac{1}{2} \cdot 1\frac{5}{9}$$

8.
$$2\frac{6}{7} \cdot (-7)$$

9. On average, people spend $\frac{1}{4}$ of the time they sleep in a dream state. If Maxwell slept 10 hours last night, how much time did he spend dreaming? Write your answer in simplest form.

Multiply. Write each answer in simplest form.

10.
$$5 \cdot \frac{1}{8}$$

11.
$$4 \cdot \frac{1}{8}$$

12.
$$3 \cdot \frac{5}{8}$$

13.
$$6 \cdot \frac{2}{3}$$

14.
$$\frac{2}{5} \cdot \frac{5}{7}$$

15.
$$\frac{3}{8} \cdot \frac{2}{3}$$

10.
$$5 \cdot \frac{1}{8}$$
 11. $4 \cdot \frac{1}{8}$ 12. $3 \cdot \frac{5}{8}$ 13. $6 \cdot \frac{2}{3}$ 14. $\frac{2}{5} \cdot \frac{5}{7}$ 15. $\frac{3}{8} \cdot \frac{2}{3}$ 16. $\frac{1}{2} \cdot \left(-\frac{4}{6}\right)$ 17. $-\frac{5}{6} \cdot \frac{2}{3}$ 18. $7\frac{1}{2} \cdot 2\frac{2}{5}$ 19. $6 \cdot 7\frac{2}{5}$ 20. $2\frac{4}{7} \cdot \frac{1}{6}$ 21. $2\frac{5}{8} \cdot 6\frac{2}{3}$

17.
$$-\frac{5}{6} \cdot \frac{2}{3}$$

18.
$$7\frac{1}{2} \cdot 2\frac{2}{5}$$

19.
$$6 \cdot 7\frac{2}{5}$$

20.
$$2\frac{4}{7} \cdot \frac{1}{6}$$

21.
$$2\frac{5}{8} \cdot 6\frac{2}{3}$$

22.
$$\frac{2}{3} \cdot 2 \frac{91}{4}$$

23.
$$1\frac{1}{2} \cdot 1\frac{5}{9}$$

24.
$$7 \cdot 5\frac{1}{8}$$

22.
$$\frac{2}{3} \cdot 2^{\frac{91}{4}}$$
 23. $1^{\frac{1}{2}} \cdot 1^{\frac{5}{9}}$ 24. $7 \cdot 5^{\frac{1}{8}}$ 25. $3^{\frac{3}{4}} \cdot 2^{\frac{1}{5}}$

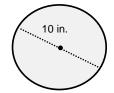
- 26. Sherry spent 4 hours exercising last week. If $\frac{5}{6}$ of the time was spent jogging, how much time did she spend jogging? Write your answer in simplest form.
- Fig. 1. An overtly blocked assignment. Every problem shares the same instructions and the same format, making it obvious to students that every problem can be solved by the same procedure. In this assignment, in fact, students can solve the word problems without reading the words.

Lesson 16-1

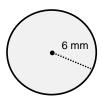
SpringBoard (p. 177)

1. Find the circumference of each circle below. Use 3.14 for π .

a.



b.



- 2. The diameter of a pizza is 14 inches. What is the circumference of the pizza? Tell what value you used for π .
- 3. The radius of a circular mirror is 4 centimeters. What is the circumference of the mirror? Tell what value you used for π .
- 4. The radius of a circular garden is 28 feet. What is the circumference of the garden? Tell what value you used for π .
- 5. Find the diameter of a circle if C = 78.5 feet. Use 3.14 for π .
- 6. Find the radius of a circle if C = 88 yards. Use 22/7 for π .

Fig. 2. A covertly blocked assignment. Although the problems in this assignment demand a variety of tasks (e.g., use diameter to find circumference, or use circumference to find radius), every problem relates to circumference.

Fair Game Review

Big Ideas (p. 219)

Write the decimal as a fraction or mixed number in simplest form.

- 42. 0.46
- 43. 0.31
- 44. 2.2
- 45. 4.32

Simplify the expression.

46. 4x + 3 - 9x

47. 5 + 3.2n - 6 - 4.8n

48. 2y - 5(y - 3)

- 49. $-\frac{1}{2}(8b+3)+3b$
- 50. Ham costs \$4.48 per pound. Cheese costs \$6.36 per pound. You buy 1.5 pounds of ham and 0.75 pounds of cheese. How much more do you pay for the ham?

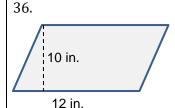
Common Core Review

Glencoe (p. 630)

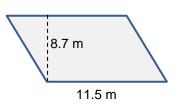
35. A frame for a collage of pictures is in the shape of a trapezoid. The two bases are 15 in. and 20 in. The height of the trapezoid is 12 inches. What is the area enclosed by the frame?

Find the area of each parallelogram. Round to the nearest tenth if necessary.

37.



5 cm



38.

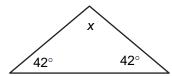
Fig. 3. Review assignments with small blocks. Most texts include periodic review assignments comprised heavily of small blocks. The assignment in the upper panel, for instance, includes two blocks of four problems each.

7.9 cm

Standardized Test Prep

Holt McDougal (p. 353)

11. What is the unknown angle measure in degrees?



- 12. The number of members of a club increases by 40. If there were 60 members before the increase, how many members are there now?
- 13. An antiques dealer bought a chair for \$85. The dealer sold the chair at her shop for 45% more than what she paid. To the nearest whole dollar, what was the price of the chair?
- 14. What is the value of the expression $-4x^2y y$ for x = -2 and y = -5?

Lesson 90 Practice

Saxon (p. 621)

- 13. If the perimeter of a square is 1 ft., what is the area of the square in square in.? (20)
- 14. a. Convert 1.75 to a fraction.
- (48) b. Convert 1.75 to a percent.
- 15. If sales tax rate is 6%, what is the total price of a \$325 printer including sales tax? (46)
- 16. Multiply. Write the product in scientific notation. (6 x 10^4)(8 x 10^{-7}) (83)
- 17. A cereal box is 8 inches long, 3 inches wide, and 12 inches tall. (67, 70) a. What is the volume of the box?
 - b. What is the surface area of the box?

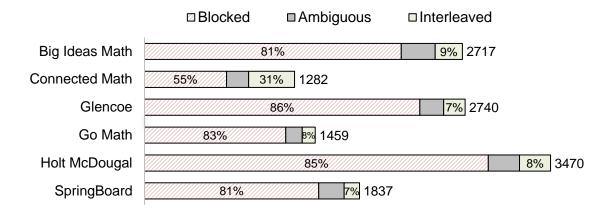


- 18. a. Find the circumference of the circle.
- (65, 82) b. Find the area of the circle.



Fig. 4. Interleaved assignments. Consecutive problems require different strategies, which forces students to choose an appropriate strategy on the basis of the problem itself. In the Saxon excerpt, the parenthetical values indicate the lesson(s) in which the relevant concepts were introduced.

A All Problems



B Review Assignments Only

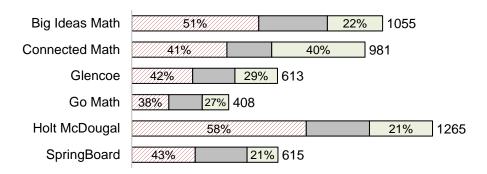


Fig. 5. Number of problems and the percentage of blocked and interleaved practice in six textbooks. (A) Each textbook is mostly blocked. (B) Even the review assignments included many small blocks, as illustrated by the sample assignments shown in Fig. 3.